Hertentamen Statistiek voor KI/Inf/BMT (Külske)

Friday 11 April 2008, 14:00

All books, written notes, and all calculators allowed.
Cell phones and laptops not allowed.

1. Suppose that a random variable Y has the pdf (density function) $f_Y(y) = \frac{1}{4}e^{-\frac{y}{4}}$ for $y \geq 0$ and zero else.

   a) What is the pdf of the new random variable $\frac{Y}{4}$?

   b) What is the pdf of $4Y$?

   c) What is the pdf of Y^4?

   d) What is the pdf of 4^{-Y}?

2. You flip a (fair) coin 12 times. For a fair coin heads and tails are equally likely.

   a) How big is the probability to get precisely 4 heads?

   b) How big is the probability to get not more than 4 heads?

   c) How big is the conditional probability to get precisely 4 heads if you already know that you did not get less than 4 heads?

   d) How big is the conditional probability to get precisely 4 heads if you already know that you did not get more than 4 heads?

3. **a)** You flip a coin 12 times and you get 4 heads and 8 tails.

   What is the P-value under the assumption of fairness?

   Can you reject the null-hypothesis of a fair coin at the level of 5 percent?

   b) Now you flip a coin 1000 times and you get 550 heads.

   Can you reject the null-hypothesis of a fair coin at the level of 5 percent?

   Use the Normal approximation to the Binomial distribution!

4. Consider a normal random sample $X_1, \ldots, X_n$ with expectation μ and variance σ^2, where the variance σ^2 is unknown.

   a) Suppose that a test for the mean μ is made, based on one random sample. Consider the following statements:

   (S1) We reject the null Hypothesis $\mu = 0$ against the alternative $\mu < 0$ to the level $\alpha = 0.01$.

   (S2) We reject the null Hypothesis $\mu = 0$ against the alternative $\mu \neq 0$ to the level $\alpha = 0.01$.

   Is it true that (S2) follows from (S1)?
   Is it true that (S1) follows from (S2)?

   b) Now two different samples from this population are taken. A 90-percent confidence interval for μ is constructed with the first sample, and a 99-percent confidence interval is constructed with the second. Will the 99-percent confidence interval necessarily be longer than the 90-percent confidence interval or will the 90-percent confidence interval necessarily be longer than the 99-percent confidence interval or can we make no general statement?

   Explain!

5. 2 percent of the population has a disease. There are three medical tests developed to detect it. All of these three tests classify 95 percent of the healthy people as healthy, and classify 95 percent of the sick people as sick. Let us assume that the tests are independent.

   How big is the chance that a person is sick if all three tests are positiv?